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THE ASSESSMENT OF CONTACT FATIGUE ENDURANCE OF RAILS IN REAL OPERATING CONDITIONS

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ABSTRACT

The article deals with the technique allowing the assessment of rail endurance with respect to contact fatigue durability with consideration of intensity of lateral wear of railhead. The method provides for taking into account the entire diversity of the rolling stock moving at different speeds on the certain railroad section, which is characterized by the curvature radius, the elevation of the outer rail, the upper structure of the railway track, and other operational factors. The method is based on the use of the program to assess the probabilistic characteristics of the interaction processes between the track and the rolling stock.

The rail wear caused by contact fatigue defects, as well as due to durability determines mainly the service life of the railway track between major repairs.

***KEYWORDS:** Contact Fatigue Durability, Lateral Wear Of The Railhead, Rail Durability, Operating Conditions & Passed Through tonnage*

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Original Article

INTRODUCTION

As shown by the results of experimental and theoretical works (Tarnopolsky, Shklyar, 1970; Verigo, Kogan, 1986; Kogan, Abdurashitov, 2014), the value of the resultant vertical and lateral loads acting from the wheel on the rail, raised to the power of γ(γ = 4÷5), can be taken as a measure of damage to the rails, caused by contact fatigue defects in a single loading cycle μ:

$$\mu = R^{\gamma} = (Q^2 + Y_L^2)^{\gamma/2} \tag{1}$$

where $Q^2 + Y_L^2 = R^2$ is the square of the resultant vertical Q and the lateral Y_L loads transmitted from the railroad carriage wheel to the rail.

The vertical and lateral loads acting on the railway track cross-section at the time of wheel passing through this section are random variables.

When random force Q passes through the cross section of the railway track, the mean square of this force is determined by the expression

$$\langle Q^2 \rangle = \langle Q \rangle^2 + \sigma_Q^2 \tag{2}$$

where $\langle Q \rangle$ is the mean of the vertical force Q; σ_Q is the standard deviation of the Q from the mean

value Q.

By analogy, the average square of the lateral force Y_L is determined by the expression:

$$\langle {Y_L}^2 \rangle = \langle Y_L \rangle^2 + \sigma_{Y_L}^2 \tag{3}$$

When passing the entire variety of trains through a certain section of the railway track, the axis of the railroad carriage transmits vertical forces with the following statistical characteristics to the rail

Where $\langle Y_L \rangle$ is the mean value of the vertical force Y_L;

σ_{Y_L} is the standard deviation of Y_L from the mean value $\langle Y_L \rangle$.

$$\langle Q \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle Q_i \rangle\,; \tag{4}$$

$$\sigma_Q^2 = \frac{1}{N} \sum_{i=1}^{N} \langle \sigma_{Q_i}^2 \rangle, \tag{5}$$

Where i is the number of the wheel passing through a certain section of the railway track;

$\langle Q_i \rangle$ is the average value of the vertical load Q_i transmitted from the i-th wheel to the rail;

σ_{Q_i} is the standard deviation of the vertical force Q_i transmitted from the i-th wheel to the rail from the average value $\langle Q_i \rangle$;

N is the total number of wheels passing through a certain section of the railway track during the day.

By analogy with (4) and (5), the statistical characteristics of the random lateral force acting from the axes of the railroad carriages are defined as follows:

$$\langle Y_L \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle Y_{L_i} \rangle \tag{6}$$

$$\sigma_{Y_L}^2 = \frac{1}{N} \sum_{i=1}^{N} \langle \sigma_{Y_{L_i}}^2 \rangle \tag{7}$$

Where $\langle Y_{L_i} \rangle$ is the average lateral force Y_{V_i} transmitted from the i-th wheel on the rail;

$\sigma_{Y_{L_i}}$ is the standard deviation of the lateral force Y_{V_i} transmitted from thei-th wheel to the rail from the average value $\langle Y_{L_i} \rangle$

Taking into account formulas (1-7), the average damage rate of the rails by contact fatigue damages in one loading cycle is determined by the expression (Kogan, Abdurashitov, 2014):

$$\langle \mu \rangle = \left\{ \left[\frac{1}{N} \sum_{i=1}^{N} \langle Q_i \rangle \right]^2 + \left[\frac{1}{N} \sum_{i=1}^{N} \langle Y_{L_i} \rangle \right]^2 + \frac{1}{N} \sum_{i=1}^{N} \left(\sigma_{Q_i}^2 + 2K_{Q_i Y_{L_i}} \sigma_{Q_i} \sigma_{Y_{L_i}} + \sigma_{Y_{L_i}}^2 \right) + \sigma_s^2 \right\}^{\gamma/2} \tag{8}$$

Where $K_{Q_i Y_{L_i}}$ is the correlation coefficient of vertical Q_i and lateral Y_{L_i} forces transmitted from the i-th wheel to the rail;

σ_s is the standard deviation of the vertical dynamic force transmitted from the wheel to the rail and caused by random impacts of the wheel flats on the rolling surfaces of the wheels of the rolling stock

METHODS

General Description

The method to determine the standard deviation σ_s is described in the article (Kogan, 2014), and thus we will not dwell on it.

The value of $\langle\mu\rangle$ should be determined individually for the outer and inner length of rails.

In the optimal case, the maximum permissible values of defects on contact fatigue and wear and tear should be achieved simultaneously. However, it is better to remove rails due to wear before the emergence of a high degree of risk of fatigue destruction. At that, it is necessary to use a fatigue resource of rails as much as possible. For this purpose, the coefficient *K* determining rail damage by contact fatigue defects caused by intense lateral wear *I*, mm/mln ton was introduced. By definition, this coefficient is equal to the ratio of the damage rate of the rails by contact fatigue defects in one loading cycle $\langle\mu\rangle$ at a given intensity of lateral wear of the rails *I* to that in the absence of lateral wear.

Algorithm

According to the research carried out by Abdurashitov (Abdurashitov, 2002; Kogan, Abdurashitov, 2009), this coefficient is determined by the empirical formula:

$$K = \begin{cases} 1 - 43.65I + 698.2I^2 - 4493I^3 & \text{at } 0 \le I \le 0.05 \\ 0 & \text{at} I > 0.05 \end{cases} \qquad (9)$$

As noted by Abdurashitov, the beneficial effect of lateral wear on the reduction of contact fatigue is no longer valid when the longitudinal and transverse contact fatigue cracks are formed under the "shelf" in severely worn rail, when the flanges of the wheels are rolling on the "shelf". Therefore, in operation, it is necessary to carry out special measures (for example, grinding on the rail surface and creating artificial vertical wear of the railhead) to prevent such a condition of the rails.

DISCUSSION

For the subsequent presentation of the material, we will focus on the method of estimating the intensity of lateral wear of rails in real operating conditions, the basics of which are given in (Kogan, 2017).

Let us first consider the problem of determining the lateral wear of the rail under passing trains.

The wheel *i* of the railroad carriage of j design, when rolling the outer length of rails, forms the over swing b_{ij}, which is a horizontal longitudinal distance from the instantaneous center of rotation of wheel *i* of the railroad carriage of *j* design (point 0 in Fig.1) up to the touch point of the wheel and rail crest 0_1.

The over swing is determined as follows (Verigo, Kogan, 1986):

$$b_{ij} = r_j tg\psi_{ij}^r \mathrm{tg}\tau_j \approx r_j\psi_{ij}^r \mathrm{tg}\tau_j \qquad (10)$$

Where r_j is the wheel radius of the railroad carriage of *j* design;

ψ_{ij}^r is the angle of attack of wheel *i* of the railroad carriage of *j* design;

τ_j is the inclination angle to the horizon of the crest of the rolling wheel of the railroad carriage of *j* design

If the directive force $Y_{1_{ij}}$ is applied at the contact point of the rotating rolling wheel *i* of the railroad carriage of *j* design and the side surface of the outer length of rails, then a friction force occurs, defined as follows

$$T_{ij} = \mu_0 R_{ij} \tag{11}$$

Where μ_0 is the friction coefficient between the surfaces of the wheel crest and rail;

R_{ij} is the normal force at the point of contact of the crest of the wheel *i* of the railroad carriage of *j* design and the lateral surface of the rail

The normal force at the point of contact of the rolling wheel crest and the lateral surface of the rail is determined by the expression (Kogan, 1997):

$$R_{ij} = \frac{Y_{1_{ij}}}{\sin\tau_j - \mu_0 \cos\tau_j} \tag{12}$$

The friction work T_{ij}at the displacement of *ds*, where *ds* is the differential railway track, on which the friction force does the work, is determined by the equality

$$dA = T_{ij} ds \tag{13}$$

Substituting the expression (10) in (12) and taking into account the equality (11), one obtains

$$dA = \mu_0 Y_{1_{ij}} \frac{1}{\sin\tau_j - \mu_0 \cos\tau_j} ds. \tag{14}$$

Suppose that in consequence of a single passage of the rolling wheel *i* of the railroad carriage of *j* design along the rail curve, the lateral surface of the railhead loses the cross-sectional area of ΔS_{ij}^L.

Dividing both parts of equality (13) by ΔS_{ij}^L, and taking into account that $d\Omega^L = \Delta S_{ij}^L dx$ represents the differential of volume loss by metal of the outer length of rails, when i-throlling wheel couple of railroad carriage of *j* design passes the distance *dx*, one gets

$$\frac{dA}{d\Omega^L} = \frac{1}{\Delta S_{ij}^L} \mu_0 Y_{1_{ij}} \psi_{ij}^r \frac{\sec\tau_j}{\sin\tau_j - \mu_0 \cos\tau_j} \tag{15}$$

Since the ratio of the work, spent on wear, to the volume of the material lost during wear is the resistance against wear *C*, one obtains the following estimate of the cross-sectional area of the railhead lost due to lateral wear at a single pass along the curve of the i-throlling mounted wheels of railroad carriage of *j* design:

$$\Delta S_{ij}^L = \frac{1}{C} \mu_0 Y_{1_{ij}} \psi_{ij}^r \frac{\sec\tau_j}{\sin\tau_j - \mu_0 \cos\tau_j} \tag{16}$$

The concept of wear resistance is used to assess the quality of polymeric materials. With regard to rail steel, the concept of wear resistance *C** is used, which is defined as the inverse of the mass loss in the sample during wear tests.

The tests were carried out in a laboratory environment. Wear resistance was assessed during dry wear tests on samples 40 mm in diameter and 4-6 mm thick, cut near the rolling surface. The tests were carried out using Amsler wear testing machine with regard to resistance to rolling with 10% slip. Wheel steel rollers were used as a counter body. Wear was estimated by weight loss after $7 \cdot 10^4$ turns of rollers pressed to each other by a force of 0.3-0.7 kN (Zolotarsky, Rauzin, Schuur, 1976).

Lasting quality C and wear resistance C* are related linearly (Kogan, 2017), so that

$$C = 5.5 \cdot 10^{2} C^* \quad (17)$$

In formula (17), quantity C has the dimensionality of $\frac{\mathrm{T}J}{m^2} = \frac{kN}{mm^2}$, while quantity C* has the dimensionality of g^{-1}.

Formula (16) can be used to obtain the total cross-sectional area of the railhead, lost in consequence of the friction interaction of the rail with the crests of the rolling wheels of the rolling stock. Taking into account the correlation (17), one can write

$$S^L = 0.182 \cdot 10^{-2} \frac{\mu_0 N^*}{C^*} \sum_{j=1}^{M} \beta_j \frac{\sec \tau_j}{\sin \tau_j - \mu_0 \cos \tau_j} G_j^L \quad (18)$$

where

S^L is the total cross-sectional area of the railhead lost as a result of abrasion of the lateral face of the railhead by the wheel crests of the passing trains, mm^2;

N^* is the total number of railroad carriages that have passed along the given curve;

M is the total number of railroad carriage designs running along the considered section of the rail track;

β_j is the proportion of railroad carriages with design parameters of *j*, moving at specified speeds and certain axial loads, in the general variety of railroad carriages circulating on the considered railroad section;

G_j is the average value of the complex index that determines the lateral wear of the railhead when passing the railroad carriage with *j* design along the considered curve, kN.

The value of G_j^L can be calculated by the formula

$$G_j^L = \sum_{i=1}^{n_j^r} \langle Y_{1_{ij}} \psi_{ij}^r \rangle = \sum_{i=1}^{n_j^r} [\langle Y_{1_{ij}} \rangle \langle \psi_{ij}^r \rangle + K_{Y_{1_{ij}}} \psi_{ij}] \quad (19)$$

where

n_j^r is the number of mounted wheels rolling when moving along the considered curve of the railroad carriage of *j* design;

$\langle Y_{1_{ij}} \rangle$ is the average value of the directive force acting from the crest of the *i*-th rolling mounted wheels of the railroad carriage of *j* design along the outer length of rails under consideration;

$\langle \psi_{ij}^r \rangle$ is the average angle between the axis of the *i*-th mounted wheels and the normal to the longitudinal axis of the rail railway track *x* at the location of the mounted wheels;

$K_{Y_{1_{ij}}} \psi_{ij}$ is the correlation moment (covariance) of random variables $Y_{1_{ij}}$ and ψ_{ij}

Methods to determine $\langle Y_{1_{ij}} \rangle$, $\langle \psi_{ij}^r \rangle$, and $K_{Y_{1_{ij}}} \psi_{ij}$ quantities are described in (Kogan, 1997).

The lateral wear δ^L is related to the cross-sectional area of the rail lost due to wear. A corresponding graph for the P65 rail is shown in Figure.2a.

Thus, the lateral wear of the railhead δ^L in the considered curve is determined by the expression:

$$\delta^L = k_L \Phi_L\{S^L\} =$$

$$= k_L \Phi_L \left\{ \frac{0{,}182 \cdot 10^{-2} \mu_0 N^*}{C^*} \sum_{j=1}^{M} \beta_j \frac{\sec \tau_j}{\sin \tau_j - \mu_0 \cos \tau_j} \sum_{i=1}^{n_j^r} [\langle Y_{1_{ij}} \rangle \langle \psi_{ij}^r \rangle + K_{Y_{1_{ij}}} \psi_{ij}] \right\} \tag{20}$$

where

k_L is the coefficient that corrects the lateral wear of the railhead taking into account the increased impact of locomotives in traction mode

This coefficient can be taken to be equal to:

k_L = 1.05 in railroad sections with slopes of 12-15%;

k_L = 1.10 in the mountain-pass sections of the railroad.

The intensity of the lateral wear of the rail head *I*, which is included in the expression (9), and significantly affects the contact fatigue service life of the rail, is determined by the expression

$$I = \frac{d\delta^L}{dT} \tag{21}$$

Where T is the gross tonnage, at which the lateral wear of the railhead amounts to δ^L mln tons

As follows from the expression (20), the intensity of the lateral wear of the railhead, determined by the expression (21), depends on the gross tonnage passed through the railway section.

It is convenient to introduce another definition of the lateral wear intensity of the rail as the ratio of the cross-sectional area of the railhead lost in consequence of abrasion of its lateral plane by the wheel flanges of the railroad carriages passing through the railway section to the passed gross tonnage:

$$J^L = \frac{d\Delta S^L}{dT} \tag{22}$$

The value of J^L does not depend on the number of railroad carriages passed through the railway section, and taking into account (19), is determined by the expression

$$J^L = \frac{0{,}182 \cdot 10^4 \mu_0}{C^* \langle 2Q_0 \rangle \langle n_j \rangle} \sum_{j=1}^{M} \beta_j \frac{\sec \tau_j}{\sin \tau_j - \mu_0 \cos \tau_j} \sum_{i=1}^{n_j^r} [\langle Y_{1_{ij}} \rangle \langle \psi_{ij}^r \rangle + K_{Y_{1_{ij}}} \psi_{ij}] \tag{23}$$

Where $\langle 2Q_0 \rangle = 2 \sum_{j=1}^{M} \beta_j \langle Q_{0_i} \rangle$ is the average axial load of the railroad carriage running through the certain section;

$\langle n_j \rangle = \sum_{j=1}^{M} \beta_j \, n_j$ is the weighted average number of passed railroad carriage axles;

The multiplier 10^6 in the formula (23) is introduced due to the fact that the dimension of the lateral wear intensity J^L is convenient to be expressed in the units of gross mm^2/MMT (tonnage dimension in MMT).

The lateral wear intensities *I* and J^L are interrelated by a simple relation

$$I = \frac{d\delta^L}{dT} = \frac{d\delta^L}{d\Delta S^L} \cdot \frac{d\Delta S^L}{dT} = \frac{d\delta^L}{d\Delta S^L} \cdot J^L \tag{24}$$

Graphical representation of the derivative $\frac{d\delta^L}{d\Delta S^L}$ depending on the lateral wear δ^L is shown in Fig. 2b.

If neglecting the effect of lateral wear of the rail in the initial period of operation (δ<5 mm) on its service life in terms of contact fatigue durability, then, when determining the coefficient *K* in the formula (9), the value $\frac{d\delta^L}{d\Delta S^L}$can be considered constant.

In this case $\frac{d\delta^L}{d\Delta S^L}$ = 0.038 mm^{-1} = *const*.

Substituting expression for J^L defined by formula (22)into the correlation (23), one obtains the following result:

$$I = \frac{0{,}69 \cdot 10^2 \mu_0}{C^* \langle 2Q_0 \rangle \langle n_j \rangle} \sum_{j=1}^{M} \beta_j \frac{\sec \tau_j}{\sin \tau_j - \mu_0 \cos \tau_j} \sum_{i=1}^{n_j^r} [\langle Y_{1_{ij}} \rangle \langle \psi_{ij}^r \rangle + K_{Y_{1_{ij}}}] \tag{25}$$

After the lateral wear intensity of the rail *I* is determined, using formula (9), one can determine coefficient *K*, which is equal to the ratio of the rail damage rate by contact fatigue defects during one loading cycle $\langle \mu \rangle$ at the actual lateral wear rate of the rails within a given railway section, to such in the absence of lateral wear.

Taking into account the intensity of lateral wear *I*, the linear damage index of rails by contact fatigue damage in a single loading cycle during the passage of the entire variety of railroad carriages along the railroad section will be determined as follows:

$$D = \beta_1 \beta_2 \beta_3 \sum_i \sum_j \alpha_i \varepsilon_j \langle \mu_{ij} \rangle K_{ij} \tag{26}$$

where

$\langle \mu_{ij} \rangle$ is the average value of the linear damage rate of rails by contact fatigue defects in one loading cycle for a given railroad carriage moving at a given speed and axial load (*i* index) along the railway track of a given design being at a certain condition, including geometry in plan, profile, and level (*j* index);

K_{ij} is the coefficient *K*, calculated for the same conditions as $\langle \mu_{ij} \rangle$indicator;

α_i is the proportion of railroad carriages moving at specified speeds and axial load (*i* index);

ε_j is the proportion of the length of the plan and profile element (*j* index);

β_1 is the coefficient, which takes into account the effect of rail grinding on its contact fatigue service life;

β_2 is the coefficient, which takes into account the influence of multiplicity of traction, regenerative braking, and mountain-pass conditions on the rails' contact fatigue service life;

β_3 is the coefficient, which takes into account the influence of climatic conditions

In general, β_1,$\beta_2$and $\beta_3$coefficients are determined by the operating conditions of the considered railway track section; their numerical values are given in (Kogan, Abdurashitov, 2014).

The described methods allow transferring the observations' results of the rail wear due to the contact fatigue defects obtained on the experimental section of the railway track to any other section.

Indeed, at the same resource of contact fatigue durability of rails, using the linear damage accumulation rules, we have

$$n_1 D_1 = n_0 D_0 = G = const \tag{27}$$

where

n_1 is the number of carriage axes, which can be passed through section 1 until the contact fatigue service life of the rails *G* is exhausted;

D_1 is the indicator of rails' damage by contact fatigue defects at section 1 corresponding to the passage of one axle of the train;

n_0 is the number of axes of the railroad carriages passed through the experimental section before contact and fatigue durability recourse is exhausted;

D_0 is the indicator of rails' damage by contact fatigue defects on the experimental section during the passage of one axle of the train.

Taking into account that

$$\mathrm{n}_1 = \frac{\mathrm{T}_1}{\langle \mathrm{P}_1 \rangle};\ \mathrm{n}_0 = \frac{\mathrm{T}_0}{\langle \mathrm{P}_0 \rangle} \tag{28}$$

where

T_1 is the gross tonnage, which can be passed over section 1 until the rail contact fatigue life is exhausted;

$\langle P_1 \rangle$ is the average axial load at section 1;

T_0 is the gross tonnage passed through the experimental section until the life of the contact fatigue durability of the rails is exhausted;

$\langle P_0 \rangle$ is the average axial load at the experimental section of railway track

one obtains an expression that defines the gross tonnage, which can be passed through the railroad section 1.

$$\mathrm{T}_1 = \mathrm{T}_0 \frac{\mathrm{D}_0 \langle \mathrm{P}_1 \rangle}{\mathrm{D}_1 \langle \mathrm{P}_0 \rangle} \tag{29}$$

Formula (28) allows calculating the gross tonnage, which can be passed through a given railway section with the specified characteristics, if the gross tonnage passed through the experimental section, at which the contact fatigue durability of the rails has been exhausted, is known. The determining factor in the solution of this problem is the parametric relationships between D_0, D_1, $\langle P_0 \rangle$и$\langle P_1 \rangle$, which in a brief form characterize the operating conditions of the experimental and given sections of the railway track.

CONCLUSIONS

The authors offer technique to assess contact fatigue endurance of rails in real conditions of their operation combining it with estimation of rails' lateral wear intensity significantly influencing the development of contact fatigue defects at the same operating conditions.

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APPENDIX

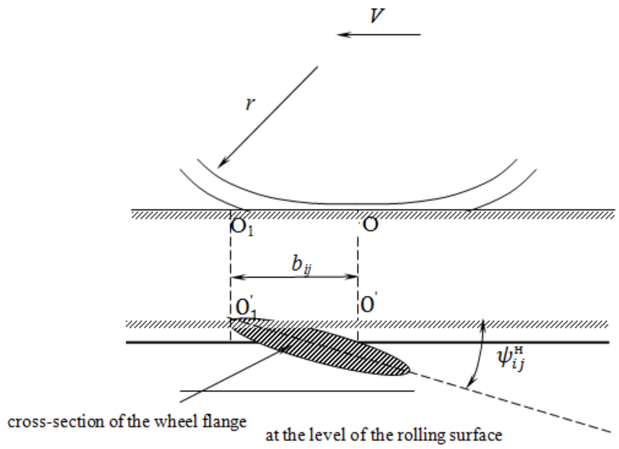
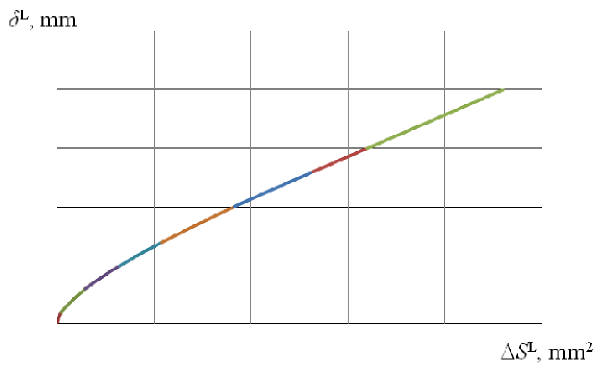
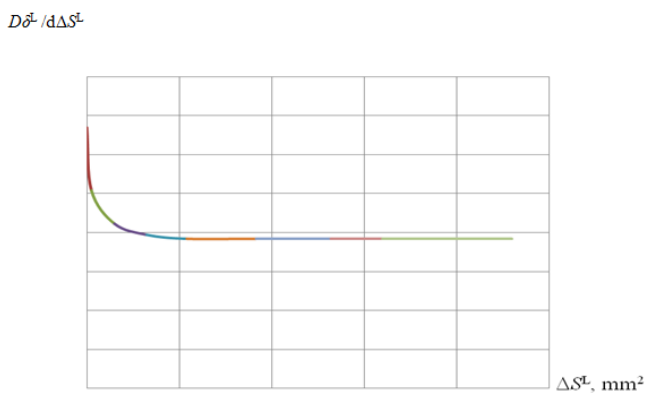


Figure 1: Formation Scheme of the Over Swing b_{ij} of the Touch Point of the Crest and Wheel $0_1'$ Relative to the Instantaneous Rotation Center of Aiming Point 0(0′)



(a)



(b)

Figure 2.The Lateral Wear of the Rail Depending on the Area of the Lost Metal

(a) Lateral wear δ^L depending on the area of the lost metal ΔS^L for rail R65;
(b) Derivative of the lateral wear $d\delta^L / d\Delta S^L$ depending on the area of the lost metal ΔS^L for rail R65;